

Lecture 12A: Neural Nets

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Jeff Hinton published a paper in 2012 which showed the application of neural nets successfully on recognizing and classifying pictures.

- models of problem solving
- models of learning

Basic Biologically Inspired

- Neural Nets

Naive Neurobiology

World's simplest NET problems.

Hill Climbing

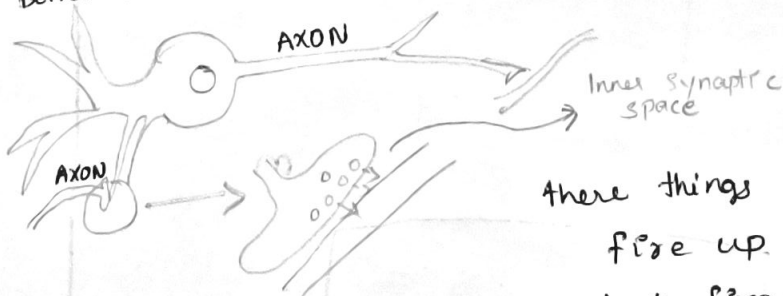
Threshold Trick

Sigmoid Trick.

Let's study Some NEUROBIOLOGY



DENDRITE TREE



Synaptic Weight.

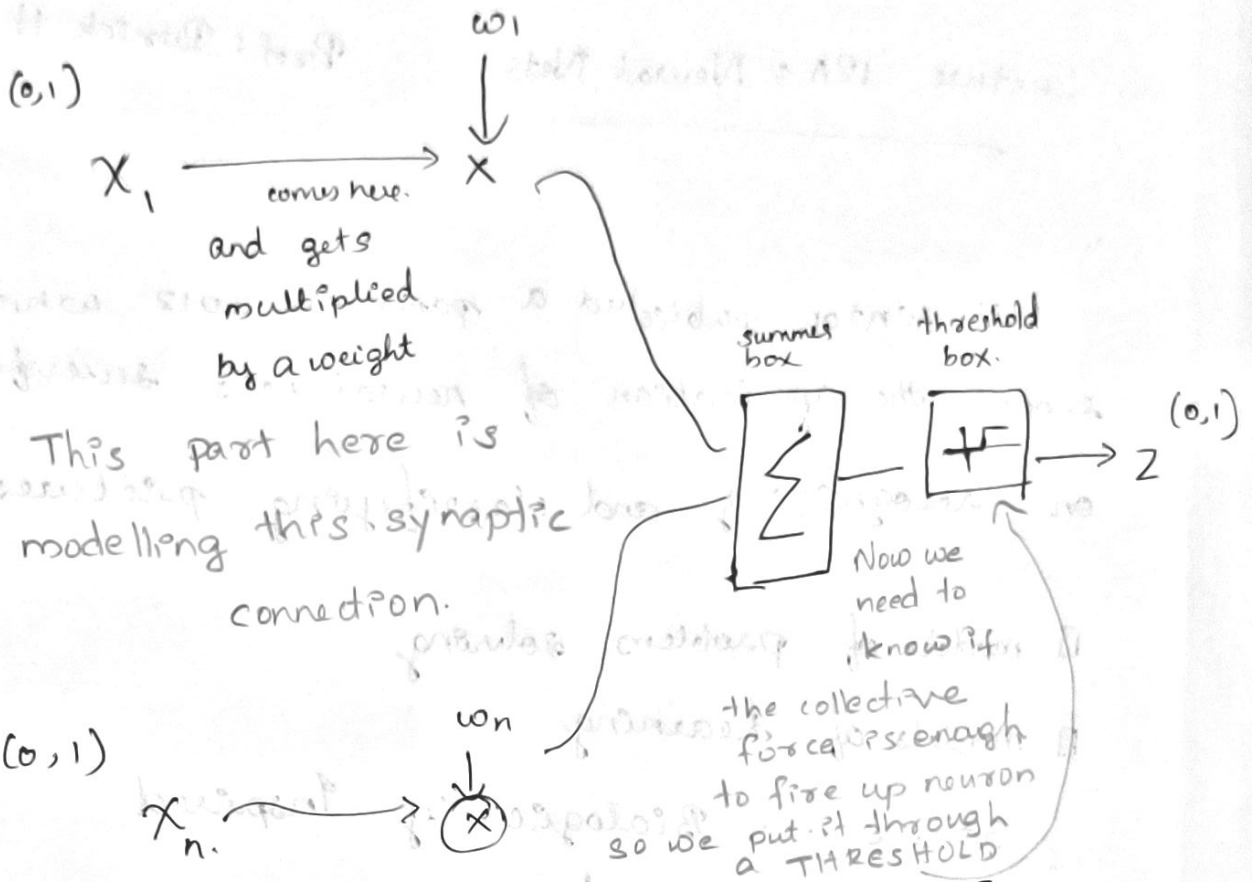
strength or amplitude of a connection b/n. two nodes.

these things either they fire up $\equiv 1$

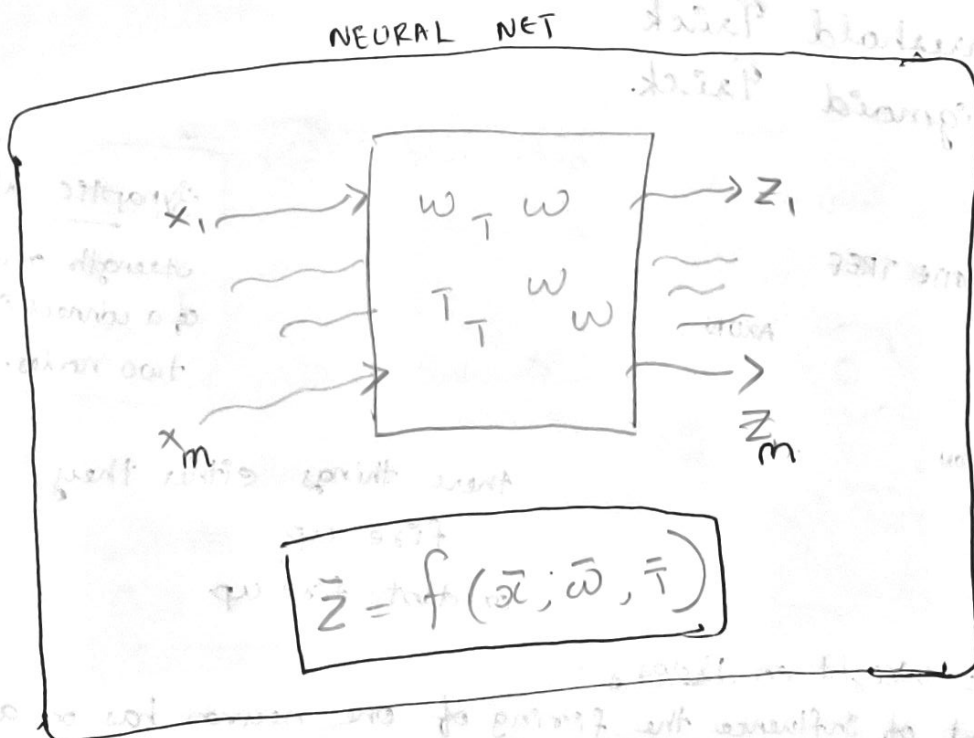
or don't fire up $\equiv 0$

synaptic weight in Biology =

Amount of influence the firing of one neuron has on another.



we need to look at the collective force of these inputs.



so, when we train a neural net, all we are going to be able to do is adjust those weights and thresholds so that what we get out is what we want.

Hence, a neural net is a function approximator:

$$\bar{z} = f(\bar{x}, \bar{w}, \bar{t})$$

$$\left(1 \cdot \frac{96}{500} + 1 \cdot \frac{96}{500} \right) = 0.384$$

$$\bar{d} = g(\bar{x})$$

desired output

How well are we doing?

This can be figured by comparing the desired value with actual output value.

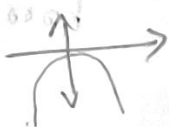
Performance function, $P = \|\bar{d} - \bar{z}\|$

but the abs value graph looks like



this mathematically inconvenient

so we square it and multiply with -1



now all we need to do, is

Maximize $P = -\|\bar{d} - \bar{z}\|^2$

maximize,

$$P = -\|\hat{d} - \bar{z}\|^2$$

lets take partial derivative

here we are trying to take our weights and thresholds, and trying to maximize our performance.

$$\Delta \hat{w} = \sigma \left(\frac{\partial P}{\partial w_1} i + \frac{\partial P}{\partial w_2} j \right)$$

σ = rate constant.

~~derivative~~

for example

$$\text{if } \frac{\partial P}{\partial w_2} > \frac{\partial P}{\partial w_1}$$

it tells us, we need to move towards w_2 .
 w_2 has more influence on P .

Now we have a problem: ☹️

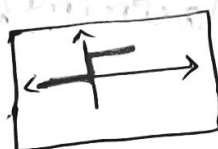
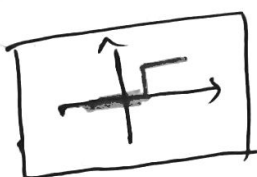
look at what P is, it has a threshold function, which is discontinuous hence not differentiable. here, how annoying! Lets fix those threshold box.

lets take $w_0 = T$ connected to -1

Step 1

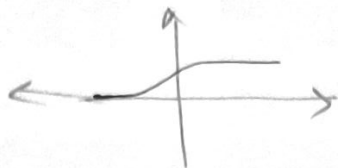
original threshold box.

new threshold box.



Step 2.

lets remove the step function.

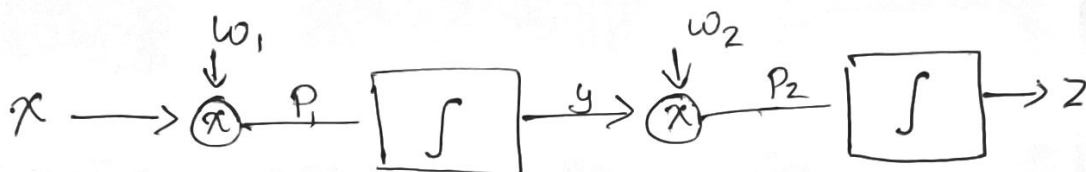


$$= \frac{1}{1 + e^{-\alpha}}$$

$\alpha = \text{input}$.

sigmoid
function

finally, life is mathematically convenient.



$$P = \frac{1}{2} (\bar{a} - \bar{z})^2$$

$$\frac{\partial P}{\partial w_2} = \frac{\partial P}{\partial z} \frac{\partial z}{\partial w_2} \rightarrow \frac{\partial z}{\partial p_2} \cdot \frac{\partial p_2}{\partial w_2}$$

MAGIC
OF
CHAINRULE

$$\frac{\partial P}{\partial w_1} = \frac{\partial P}{\partial z} \frac{\partial z}{\partial p_2} \frac{\partial p_2}{\partial w_2} \cdot \frac{\partial p_2}{\partial y} \frac{\partial y}{\partial p_1} \frac{\partial p_1}{\partial w_1}$$

~~rewriting~~ re writing for convenience
and use the following.

$$\frac{\partial P}{\partial z} = \frac{\partial}{\partial z} \left(\frac{1}{2} (\bar{a} - \bar{z})^2 \right) =$$

$$\frac{\partial p_2}{\partial w_2} = \frac{\partial}{\partial w_2} y w_2 = y$$

$$p = \frac{1}{1 + e^{-\alpha}} = \frac{\partial p}{\partial \alpha} = \frac{\partial (1 + e^{-\alpha})^{-1}}{\partial \alpha}$$

$$= - (1 + e^{-\alpha})^{-2} \cdot e^{-\alpha} \cdot (-1)$$

$$= e^{-\alpha} (1 + e^{-\alpha})^{-2}$$

$$= p(1 - p)$$

even if we add more inputs, its still linear in depth, as the derivative is already calculated.